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ON THE ANALYSIS OF ELECTRIC EFFECTS IN AN IONIZED MULTICOMPONENT GAS AROUND CONDUCTING BODIES. THE METHOD OF SEPARATION

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M. S. BENILOV and G. A. TIRSKII
(Moscow)

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Formulation of the hydrodynamic problem of determining electrical effects in a moderately ionized gas surrounding conducting bodies is investigated in the case when the characteristic Debye radius is considerably smaller than the characteristic dimension of the body in the gas stream. A new statement of the problem is proposed which considerably simplifies its numerical, as well as analytical investigation. A method of separation is formulated which makes possible the determination of concentrations and diffusion fluxes of ionized components throughout the region occupied by gas, including the Debye layer, by solving the initial problem in a quasi-neutral formulation. The electric field throughout the volume of gas is then determined by the solution of a linear differential equation with known coefficients. It is shown that in a particular region of potentials of the body surface the solution obtained in this manner is uniformly valid.

The one-dimensional problem of the electric probe in a weakly ionized three-component gas is investigated as an example. The method of separation applied to the considered problem yields linear equations for the concentration of charged particles and for the electric field. These equations are solved in quadratures. Explicit formulas are derived for linear sections of the ion and electron characteristics.

The interpretation of measurements obtained by probes in dense plasma under various conditions of flow around it has recently attracted considerable interest in hydrodynamic problems related to the determination of electrical parameters around conducting bodies (see, e.g., [1 —3]. The hydrodynamic model is applicable through the whole volume occupied by an ionized gas on the fundamental assumption that the particle free path λ is considerably shorter than all characteristic dimensions of the problem, in particular the Debye layer thickness λ_d, and that the particle drift velocity is considerably lower than the thermal velocity. The last condition means that the work of the electric field E over the particle free path is considerably smaller than that of heat energy kT, i.e. $eE\lambda \ll kT$. In the considered here case of the Debye collision layer that condition is satisfied, at least, when the wall potentials are $\varphi_w \sim kT/e$, since in that layer the electric field is of the order of $E \sim \varphi_w / \lambda_d \sim kT / e\lambda_d$. Taking into account the inequality $\lambda \ll \lambda_d$ we obtain $eE\lambda \sim e\,(kT / e\lambda_d)\lambda \ll kT$. Note that condition $\varphi_w \sim kT/e$ is satisfied in a fairly wide range of potentials. For instance, the saturation current on a probe in a weakly ionized

plasma is already reached at a potential of a few kT/e on the body surface.

The considered problem is nonlinear and, when the Debye radius is small in comparison with the characteristic dimension of the body (which is usually the case), it contains a small parameter ε equal to the square of the ratio of the characteristic Debye radius to the characteristic dimension of the body. This parameter appears at the term which defines the electric field in the Poisson equation. As shown in [2], the direct numerical solution of a problem of this kind in the initial formulation presents, even in the simplest case, considerable difficulties.

The transformation proposed here results in a considerable simplification of the numerical as well as of the analytic investigation of this electrohydrodynamic problem. The transformation consists of extracting from the Poisson equation components of the second order equation for the electric field using the equations of mass transfer of components. Substituting in the electrohydrodynamic system this equation for the Poisson equation stabilizes the numerical iteration algorithms in which at each iteration the hydrodynamic field determined in the preceding iteration is used for solving the above equation for the electric field and, then, the obtained in this manner electric field distribution is used for determining the new hydrodynamic field, in particular the concentration of ionized components. Note that this iteration algorithm which involves separate determination of the electric and hydrodynamic field parameters at each iteration is unstable when applied to the problem in its initial formulation. A transformation of this type was used in [4] for deriving numerical solutions in the case of a weakly ionized gas.

The proposed transformation makes it possible to simplify also the asymptotic solution of the problem at the limit $\varepsilon \to 0$. It is shown below that in a specific region of potentials of the body surface a uniformly valid solution of the problem can be obtained from equations that are asymptotically exact throughout the flow region, as $\varepsilon \to 0$. From this point of view the proposed iteration algorithm yields in this case already in the first iteration an asymptotically exact solution.

This device, here called the method of separation, is used below for solving tne one-dimensional problem of the electric probe in a moving or quiescent weakly ionized three-component gas of constant or variable properties. The method of separation makes possible the linearization of this problem and the derivation of explicit formulas for the linear sections of the ion and electron characteristics.

1. Statement of the problem. We shall investigate the steady flow of a chemically reacting partly ionized multicomponent heat conducting mixture of gases with different diffusion properties surrounding conducting bodies in the absence of external magnetic field. The magnetic field induced by internal currents is small as a relativistic effect. For simplicity we assume that the ionization of gas is moderate, i.e. sufficiently small for the presence of ionized components not to affect to any extent the flow of neutral components (complications arising in the extension to the general case are considered at the end of Sect. 2). This means that the order of magnitude of the degree of ionization must not exceed a few percent. Hence the fields of all quantities related to the gas as a whole, such as temperature, pressure, and mean mass velocity and, also, the fields of concentration and neutral component diffusion fluxes, can be determined by solving the corresponding problem without allowance for ionization,

and as formulated here, are assumed known. The problem, thus, consists of the determination of concentration and diffusion fluxes of ionized components, and of the electric field. For simplicity the temperature of all components is assumed to be the same.

We assume the determining system to consist of equations of conservation of mass of ionized components, the Stefan —Maxwell equations for ionized components [5], and of Maxwell equations for the electric field. In dimensionless form this system is

$$\mathrm{Re}\,\rho v\,\nabla\left(\frac{x_i}{m}\right)+\operatorname{div}\mathbf{J}_i^{\circ}=D_i w_i^{\cdot}\quad(i=1,\ldots,M) \tag{1.1}$$

$$\nabla x_i=\sum_{k=1}^{N}\Delta_{ik}\left(x_i\mathbf{J}_k^{\circ}-x_k\mathbf{J}_i^{\circ}\right)+(c_i-x_i)\nabla\ln p+x_i\Sigma_i^{T}\nabla\ln T+ \tag{1.2}$$

$$\left(x_iz_i-c_i\sum_{k=1}^{M}x_kz_k\right)\frac{\mathbf{E}^{\circ}}{T}\quad(i=1,\ldots,M)$$

$$\varepsilon\operatorname{div}\mathbf{E}^{\circ}=n\sum_{k=1}^{M}x_kz_k,\quad\mathbf{E}^{\circ}=-\nabla\varphi^{\circ} \tag{1.3}$$

$$c_i=\frac{m_i}{m}x_i,\quad\Delta_{ij}=\frac{a_{ij}}{n},\quad m=\sum_{k=M+1}^{N}x_km_k,\quad\mathbf{J}_i^{\circ}=\frac{La_0}{n_0}\mathbf{J}_i$$

$$\mathbf{E}^{\circ}=\frac{eL}{kT_0}\mathbf{E},\quad\Sigma_i^{T}=\sum_{k=1}^{N}\frac{x_ka_0a_{ik}}{\rho_0\rho}\left(\frac{D_k^{T}}{c_k}-\frac{D_i^{T}}{c_i}\right)$$

$$\mathrm{Re}=a_0v_0L,\quad D_i=\frac{w_{i0}^{\cdot}L^2a_0}{n_0},\quad\varepsilon=\frac{kT_0}{4\pi n_0e^2L^2}$$

Here and below c_i and x_i are the mass and molar concentrations, respectively; $\mathbf{J}_i$, $\mathbf{E}$, and D_i^T are, respectively, the dimensional numerical diffusion flux, electric field, and the coefficient of thermal diffusion; z_i is the charge number; ρ, v, $w_i^{\cdot}$, T, p, a_{ij} and n are, respectively, the dimensionless density, mean mass velocity, numerical formation rate of the i-th component in a unit volume as the result of chemical reactions, temperature, pressure, drag coefficients, and total numerical concentration, all normalized with respect to related characteristic values (the latter denoted by the zero subscript); m_i is the mass of a particle of the i-th kind normalized with respect to the quantity ρ_0 / n_0; k is the Boltzmann constant; e is the electron charge; L is the characteristic scale of the problem; N is the total number of components in the mixture; the first M numbers are assigned to ionized components.

The boundary conditions for the system of Eqs. (1.1) — (1.3) are formulated as follows. At the body surface which is assumed to be perfectly absorbing and catalytic, the concentrations of ionized components vanish

$$x_i=0\quad(i=1,\ldots,M) \tag{1.4}$$

Away from the body in the stream the concentration of ionized components tends to that in the unperturbed gas

$$x_i \to x_{i\infty} \quad (i = 1; \ldots, M) \quad \left(\sum_{k=1}^{M} x_{k\infty} z_k = 0\right) \tag{1.5}$$

(Owing to the asymptotics of the considered equations, the unperturbed gas at infinity is electrically neutral [6]).

To close the problem we need one more boundary condition for Eq. (1.3), namely, to specify (explicitly or implicitly) the potential of the body surface relative to infinity. This boundary condition is usually selected on grounds of convenience, taking into account the properties of the algorithm chosen for solving the problem.

Thus, the electric current density distribution at the surface was specified, [7], in [8] it was the ratio of densities of the ion and electron diffusion fluxes, the magnitude of the ion diffusion flux in [9], intensity of the electric field at the body surface in [4], and in [10] it was the body surface potential relative to infinity. In problems of flow of nonionized gas around nonconducting bodies the condition of absence of current to the wall was specified in [11], which corresponds to the so-called problem with the floating potential.

For reasons that will become clear later, it is convenient in this problem to specify this boundary condition formally as the linear combination of diffusion fluxes of ionized components to the body surface (the zero subscript at dimensionless diffusion fluxes are omitted)

$$-\sum_{j=1}^{M} z_j J_{jy} \sum_{k=M+1}^{N} x_k \Delta_{kj} = \alpha \quad \text{for} \quad y = 0 \tag{1.6}$$

where α is a function specified at the body surface and y is a coordinate normal to that surface.

The latter condition can be considered as an extension of the boundary condition used earlier in [6] in the numerical solution of equation of the Debye layer. Having solved problem (1.1) — (1.6) we can determine the surface potential distribution that corresponds to the given function α. Note that the boundary condition (1.6) has no physical meaning on its own. Its introduction is a formal device for simplifying the subsequent analysis of the problem. In the case of one-dimensional problems parameter α at the last stage of solution is expressed in terms of surface potential, and all results are reduced to conventional form which includes the voltampere characteristic.

2. The electric field equation. For $\varepsilon \ll 1$, which is a typical situation, Eq. (1.3) is inconvenient for determining the electric field $\mathbf{E}^\circ$, since the term with that field contains the small parameter. When the iterative numerical algorithm is used for obtaining two separate solutions of the equation at each iteration, the iteration process is divergent because the presence in the right-hand side of the eqution of the large parameter ε^{-1} that leads to the unavoidable error in the calculation of the sum $x_1 z_1 + \ldots + x_M z_M$ in the preceding iteration. To overcome this difficulty it is necessary to solve simultaneously linearized equations at every iteration which requires considerable computer time. For example, the determination of electrical effects in the neighborhood of a conducting sphere in a quiescent weakly ionized gas was carried out in [10] by solving simultaneously the linearized equations by matrix runs on the computer.

To avoid the indicated difficulty it is necessary to derive for the electric field a new equation which should be unaffected by perturbations of ionized component concentration. For this we eliminate in the determining system of equations the concentration of one of the ionized components (e.g., the number M component). We define in Eq. (1.3) the quantity x_M as

$$x_M = \frac{1}{z_M}\left(\frac{\varepsilon}{n}\operatorname{div}\mathbf{E}^\circ - \sum_{k=1}^{M-1} x_k z_k\right) \tag{2.1}$$

and substitute this expression into formula (1.2) with $i = M$. Eliminating in the obtained formula ∇x_k $(k = 1, \ldots, M-1)$ using the respective transport equations we obtain for the electric field the equation

$$\frac{\varepsilon}{n}\nabla\operatorname{div}\mathbf{E} + \frac{\varepsilon}{n}\operatorname{div}\mathbf{E}\left(\frac{\mathbf{E}}{T}\sum_{k=1}^{M} c_k z_k + \nabla\ln T\right) - \frac{\mathbf{E}}{T}\sum_{k=1}^{M} x_k z_k^2 = \sum_{j=1}^{N}\sum_{k=1}^{N}\Delta_{kj}x_k\mathbf{J}_j(z_k - z_j) + \sum_{k=1}^{M} c_k z_k \nabla\ln p + \sum_{k=1}^{M} x_k z_k \Sigma_k^T \nabla\ln T \tag{2.2}$$

$$\mathbf{E} = -\nabla\varphi^\circ$$

The determining system of equations thus contains now Eqs. (1.1), Eqs. (1.2) with $i \neq M$, and Eqs. (2.2). The quantity x_M which appears in the equations must be replaced by its expression (2.1). The corresponding transformation of boundary conditions (1.4) – (1.6) for this formulation of the problem is obvious:

at the body surface

$$x_i = 0 \quad (i = 1, \ldots, M-1), \qquad -\sum_{j=1}^{M} z_j J_{jy} \sum_{k=M+1}^{N} x_k\Delta_{kj} = \alpha \tag{2.3}$$

$$\operatorname{div}\mathbf{E} = 0$$

and away from the body

$$x_i \to x_{i\infty} \quad (i = 1, \ldots, M-1), \quad \operatorname{div}\mathbf{E} \to 0 \tag{2.4}$$

The Stefan – Maxwell equations in the case of a weakly ionized three-component gas assume the form of the Fick law. By substituting these into the equations (1.1) of conservation of mass of components, it is possible to eliminate from the direct solution of the problem the diffusion fluxes. In that case the new equation for the electric field is derived by combining the equations of ion and electron diffusion [4], and not the equations of transport. The use of that equation instead of the input Poisson equation makes possible the separation of linearized equations at each step of the iteration process, and solve these (in the case of one-dimensional problems) by a scalar run through. Examples of numerical algorithm construction were considered in [4]. We shall only mention here that it is convenient to begin the iteration process by solving the equation for the electric field, specifying as the initial approximation for the ion and electron concentration the quasi-neutral solution (see Sect. 3). The proposed algorithm does not require any considerable computer time, and is effective in a wide

range of values of parameter ε, including its very low values.

Note that Eq. (2.2) is valid also in the general case of multicomponent mixtures ionized to an arbitrary degree. It must then be included in the over-all hydrodynamic system with ponderomotive forces, and the Stefan–Maxwell equations (1.2) must be written for all components of the mixture.

3. Asymptotic analysis of the problem in the case of small Debye radii. Equation (2.2) contains a small parameter at higher derivatives, and the problem stated above may as a whole be classified as singularly perturbed. The first term of the external asymptotic expansion in the small parameter of solution of this problem is defined by the quasi-neutral system of equations which can be obtain from the input system by setting $\varepsilon = 0$. Equation (2.1) assumes the form of the algebraic condition of quasi-neutrality, while Eq. (2.2) becomes

$$\nabla\varphi = T\sum_{j=1}^{N}\sum_{k=1}^{N}\Delta_{kj}x_k\mathbf{J}_j(z_k^* - z_j^*) + T\sum_{k=1}^{M}c_kz_k^*\nabla\ln p + \sum_{k=1}^{M}x_kz_k^*\Sigma_k^T\nabla T, \qquad z_i^* = z_i\Big(\sum_{k=1}^{M}x_kz_k^2\Big)^{-1} \tag{3.1}$$

where the zero superscript at the dimensionless potential has been omitted.

The form of remaining equations of the system remains unchanged for $\varepsilon = 0$.

Close to the surface of the body in the stream there is an inner region in which the external solution is invalid. In the considered case of zero concentration of ionized components on the body surface the existence of such region is indicated by the singularity of the external solution (3.1) at the body surface. We shall call this inner region the Debye boundary layer (DBL).

Several modifications of the method of inner and external expansions are used for the derivation of asymptotic solutions for the considered problem in a wide range of the body surface potentials [1, 2]. Here, another asymptotic method valid in the region of body surface potentials that correspond to small parameters α is developed and applied.

To simplify the exposition we limit the analysis to one-dimensional problems such as, for instance, that of hypersonic flow of ionized gas in the neighborhood of a streamline passing through the leading stagnation point of a blunt body, the problem of a conducting sphere in a gas at rest, etc.

When the concentration of M components is excluded, the system of determining equations is

$$\mathrm{Re}\,\rho v\Big(\frac{x_i}{m}\Big)' + \frac{1}{r^2}(r^2J_i)' = D_iw_i^* \quad (i = 1, \ldots, M) \tag{3.2}$$

$$x_i' = \sum_{k=1}^{N}\Delta_{ik}(x_iJ_k - x_kJ_i) + (c_i - x_i)\ln' p + x_i\Sigma_i^T\ln' T + \Big(x_iz_i - c_i\sum_{k=1}^{M}x_kz_k\Big)\frac{E}{T} \quad (i = 1, \ldots, M-1) \tag{3.3}$$

$$\frac{\varepsilon}{n}\left[\frac{1}{r^2}(r^2E)'\right]' + \frac{\varepsilon}{nr^2}(r^2E)'\left(\frac{E}{T}\sum_{k=1}^{M} c_k z_k + \ln' T\right) - \frac{E}{T}\sum_{k=1}^{M} x_k z_k^2 = \tag{3.4}$$

$$\sum_{j=1}^{N}\sum_{k=1}^{N} \Delta_{kj} x_k J_j (z_k - z_j) + \sum_{k=1}^{M} c_k z_k \ln' p + \sum_{k=1}^{M} x_k z_k \Sigma_k^T \ln' T$$

$$x_M + \frac{1}{z_M}\sum_{k=1}^{M-1} x_k z_k = \frac{1}{z_M}\frac{\varepsilon}{nr^2}(r^2E)' \tag{3.5}$$

where $r = 1 + y$ and the prime denotes differentiation with respect to y. The boundary conditions (2.3) and (2.4) now assume the form

on the body

$$x_i = 0 \quad (i = 1, \ldots, M-1) \tag{3.6}$$

$$-\sum_{j=1}^{M} z_j J_j \sum_{k=M+1}^{N} x_k \Delta_{kj} = \alpha \tag{3.7}$$

$$E' + 2E = 0 \tag{3.8}$$

and at infinity

$$x_i \to x_{i\infty} \quad (i = 1, \ldots, M-1) \tag{3.9}$$

$$r^{-2}(r^2E)' \to 0 \tag{3.10}$$

We solve Eq. (3.4) for E / T and substitute the obtained expression into the transport equation (3.3). This yields

$$x_i' = -J_i\sum_{k=1}^{N} x_k \Delta_{ik} + x_i \sum_{k=1}^{N} \Delta_{ik}^* J_k - K_{pi}\ln' p - K_{Ti}\ln' T + \tag{3.11}$$

$$\frac{\varepsilon}{nr^2}(r^2E)' x_i\left[z_i^*\left(\frac{E}{T}\sum_{k=1}^{M} c_k z_k + \ln' T\right) - \frac{m_i}{m}\frac{E}{T}\right] +$$

$$x_i z_i^* \frac{\varepsilon}{n}\left[\frac{1}{r^2}(r^2E)'\right]' \quad (i = 1, \ldots, M-1)$$

where

$$\Delta_{ij}^* = \Delta_{ij} - z_i^* \sum_{k=1}^{N} \Delta_{kj} x_k (z_k - z_j)$$

$$K_{pi} = x_i - c_i + x_i z_i^* \sum_{k=1}^{M} c_k z_k$$

$$K_{Ti} = -x_i\left(\Sigma_i^T - z_i^* \sum_{k=1}^{M} x_k z_k \Sigma_k^T\right)$$

Note that now the terms which define the contribution of the electric field to the system of Eqs. (3.2), (3.5), and (3.11) contain the coefficient ε. Assuming that the considered dimensionless quantities are of order unity, we omit these terms. Equations (3.2) remain unchanged and Eq. (3.5) assumes the form the quasi-neutrality condition

$$\sum_{k=1}^{M} x_k z_k = 0 \tag{3.12}$$

while Eqs. (3.11) become

$$x_i' = -J_i \sum_{k=1}^{N} x_k \Delta_{ik} + x_i \sum_{k=1}^{N} \Delta_{ik} * J_k - K_{pi} \ln' p - K_{Ti} \ln' T \tag{3.13}$$
$$(i = 1, \ldots, M-1)$$

Formulas (3.13) are the quasi-neutral Stefan — Maxwell relationships [11]. Their structure is the same as that of corresponding relationships for a mixture of electrically neutral components.

The input problem (3.2) — (3.10) is thus divided into two: the quasi-neutral problem (3.2), (3.12), (3.13), (3.6), and (3.9) for the determination of concentrations and diffusion fluxes of ionized components, and the problem of electric field determination using the known quasi-neutral solution. These problems can be solved successively.

To investigate the validity of such division it is necessary, first of all, to estimate the difference between the quasi-neutral solution for concentrations and diffusion fluxes of ionized components and the exact solution of the input problem. Then there is the error of the electric field determination by Eq. (3.4) due to the error in coefficients resulting from the substitution of the quasi-neutral solution for ionized components for the exact one. Hence the latter must be estimated and parameters such that would nullify its effects must be found.

First, let us consider the closeness of the quasi-neutral solution for concentrations and diffusion fluxes of ionized components to the exact one. For this it is necessary to estimate the effect of terms that are proportional to ε and omitted in the transformation of (3.5) and (3.11) to (3.12) and (3.13). These terms may be essential if the assumption about the order of the dimensionless quantities being equal unity does not hold. Becuase of this a special investigation is required in the DBL region.

Depending on the body surface potential two cases are possible: (1) for moderate surface potentials ($\varphi_w \leqslant O(\ln \varepsilon^{-1})$) the Debye layer can be defined by a single asymptotic expansion [6], and (2) for high surface potentials ($\varphi_w > O(\ln \varepsilon^{-1})$) the Debye layer has a complex structure (consisting of several boundary layers) and cannot be defined by a single asymptotic expansion [12]. For purposes of this paper of interest is the first case, and will be considered here. Unlike in[6], parameter α will not be assumed fixed.

Let us determine the order of quantities in the DBL. We denote the order of magnitude of the electric field by E_d and the characteristic scale of the DBL by δ. The order of magnitude of ionized component concentration in the DBL is also equal δ. We assume the quantity

$$\sum_{k=1}^{M} x_k'(0)\, z_k^2$$

calculated from the quasi-neutral solution is finite and nonzero.

In the DBL the first term in the left-hand side of Eq. (3.4) must be of the same order as the last, hence $\delta = O(\varepsilon^{1/3})$. The thickness of the DBL determined in this manner is considerably smaller than all characteristic dimensions of the problem, in particular smaller than the viscous boundary layer thickness.

We seek the solution of mass conservation equations (3.2) of components in the Debye layer in the form of inner asymptotic expansion of the form

$$J_i(y, \varepsilon) = J_{1i}(y_d) + \varepsilon^{1/3} J_{2i}(y_d) + \ldots \quad (i = 1, \ldots, M)$$

where $y_d = y\varepsilon^{-1/3}$ is the prolate DBL coordinate. The substitution of this expansion into (3.2) yields

$$J_i = J_{1i} + \varepsilon^{1/3}[J_{2i}(0) + y_d (D_i w_i^{\circ}(x_j = 0,\ y = 0) - 2J_{1i})] + O(\varepsilon^{2/3}) \quad (i, j = 1, \ldots, M) \tag{3.14}$$

where J_{1i} and $J_{2i}(0)$ are constants that are determined by the joining with the external expansion. In deriving this expression we assumed for definiteness that at the body surface the normal component of mean mass velocity was zero. However all subsequent reasoning is valid also in the case of blowing or suction. Note that since formulas for the diffusion fluxes in the Debye layer (3.14) contain only terms of order zero and unity with respect to y_d which have to be joined, the external solution for diffusion fluxes of ionized components remains valid also in the Debye layer with an accuracy to terms of order $\varepsilon^{2/3}$. These formulas, moreover, show that the diffusion fluxes vary across the Debye layer by a quantity of order $\varepsilon^{1/3}$. It will be readily seen that the variation of drag coefficients and of neutral component concentration over the Debye layer is also of order $\varepsilon^{1/3}$.

Equation (3.4) may be written in the form

$$\frac{\varepsilon}{n}E'' - \frac{2\varepsilon}{nr}E' - \frac{2\varepsilon}{nr^2}E + \frac{\varepsilon}{nr^2}(r^2E)'\left(\frac{E}{T}\sum_{k=1}^{M} c_k z_k + \ln' T\right) - \tag{3.15}$$

$$\underset{\varepsilon^{1/3}E_d}{\frac{E}{T}}\ \underset{\varepsilon^{2/3}E_d}{\sum_{k=1}^{M} x_k z_k^2} = \underset{\varepsilon E_d}{\sum_{j=1}^{N}\sum_{k=1}^{N} \Delta_{kj} x_k J_j (z_k - z_j)} + \underset{\varepsilon^{2/3}E_d}{\sum_{k=1}^{M} c_k z_k \ln' p} +$$

$$\underset{\varepsilon^{1/3}E_d}{\sum_{k=1}^{M} x_k z_k \Sigma_k^T \ln' T}$$

where the estimate of the value of each term in the Debye layer appears under it. From this it is possible to find the order of magnitude of the electric field in the DBL

$$E_d = \begin{cases} \alpha\varepsilon^{-1/3}, & O(\alpha) > \varepsilon^{1/3} \\ 1, & O(\alpha) \leqslant \varepsilon^{1/3} \end{cases}$$

Below we consider the case of $O(\alpha) > \varepsilon^{1/3}$. All subsequent reasoning is also applicable to the opposite case but it is then necessary to substitute in the estimates $\varepsilon^{1/3}$ for α.

It is interesting to estimate the separation of charges in the Debye layer, i. e. the ration of the right-hand side of Eq. (3. 5) to terms in the left-hand side. Since in the Debye layer that ratio is α, the separation of charges in the DBL is insignificant for small α. When α is of order unity the charge separation is also of that order.

We use the derived above estimates for simplifying problem (3. 4), (3. 8), (3. 10) for the electric field. Note that the second term in the left-hand side of Eq. (3. 15) for the Debye layer has an order of magnitude of $\varepsilon^{1/3}$ relative to the first and last terms. Hence that term can be omitted in the Debye layer. In the quasi-neutral region the order of that term relative to the last is ε. Thus the second term in the left-hand side of Eq. (3. 15) is uniformly small relative to the last and can be omitted throughout the flow region. The third and fourth terms in the left-hand side of Eq. (3. 15) are similarly uniformly small relative to the last one. Hence Eq. (3. 4) can be written, with an error that is uniform throughout the flow region, in the form

$$\frac{\varepsilon}{n} E'' - \frac{E}{T} \sum_{k=1}^{M} x_k z_k^2 = \sum_{j=1}^{N} \sum_{k=1}^{N} \Delta_{kj} x_k J_j (z_k - z_j) + \sum_{k=1}^{M} c_k z_k \ln' p + \sum_{k=1}^{M} x_k z_k \Sigma_k^T \ln' T \tag{3.16}$$

Since the second term in the left-hand side of the boundary condition is of order $\varepsilon^{1/3}$ relative to the first, it can also be omitted. Boundary condition (3. 8) then assumes the form

$$E' = 0 \quad \text{for} \quad y = 0 \tag{3.17}$$

We pass to the estimate of terms omitted in Eqs. (3. 5) and (3. 11). The left- and right-hand sides of Eq. (3. 5) are, respectively, of order unity and ε in the quasi-neutral region, and $\varepsilon^{1/3}$ and $\alpha\varepsilon^{1/3}$ in the DBL. The left-hand side of Eq. (3. 11) is of order unity in the quasi-neutral region and in the DBL, the penultimate term in the right-hand side is of order ε and $\alpha\varepsilon^{1/3}$, and the last term in the right-hand side is of order ε and α in the quasi-neutral region and in DBL, respectively. Thus the penultimate term in the right-hand side of (3. 11) is uniformly small and can be omitted throughout the flow region. The disregard of the last terms of Eqs. (3. 5) and (3. 11) results in an error of order $\varepsilon^{1/3}\alpha$ in the concentration of ionized components. The error in concentration derivatives is of order $\varepsilon^{1/3}\alpha$ in the quasi-neutral region and of order α in the Debye layer.

Thus the neglect in Eqs. (3. 5) and (3. 11) of terms proportional to ε results in a relative error of order α in the concentration of ionized components and of their derivatives in the Debye layer. This error does not affect the diffusion fluxes of ionized

components, since these quantities in (3.14) are determined for the Debye layer by the external expansion with an accuracy to terms of order $\varepsilon^{1/2}$. Hence the error in diffusion fluxes is uniformly small and of order $\varepsilon^{1/2}\alpha$. The relative error in the concentration of ionized components is uniformly small for small α.

Finally, we estimate the error consequent to the substitution in Eq. (3.16) of the quasi-neutral solution (denoted below by an asterisk) for concentrations and ionized component diffusion fluxes for the exact one. Obviously a uniformly small error of order $\varepsilon^{1/2}\alpha$ will appear in the right-hand side. Let us consider the left-hand side of that equation, namely,

$$\frac{\varepsilon}{n} E'' - \frac{1}{T}\sum_{k=1}^{M} [x_k^* E + O(\varepsilon^{1/2}\alpha) E] z_k^2$$

$$\varepsilon \qquad 1 \qquad \varepsilon^{1/2}\alpha$$

$$\alpha \qquad \alpha \qquad \alpha^2$$

where the exact solution for ionized component concentration is represented in the form of the quasi-neutral solution and the corrections are of order $\alpha\varepsilon^{1/2}$. Under each term appears the estimate of its order of magnitude in the quasi-neutral region and in the Debye layer, respectively.

The error arising in the left-hand side is evidently of order α in the Debye layer and of order $\varepsilon^{1/2}\alpha$ in the quasi-neutral region. This error is uniformly small for small α.

Thus for small ε and α the solution of the input problem (3.2)—(3.10) can be obtained in two stages. In the first stage concentrations of the ionized components and their diffusion fluxes are determined by the solution of the quasi-neutral problem (3.2), (3.12), (3.13), (3.6), (3.7), (3.9). That solution can be determined by the known methods of solving problems of gasdynamics of multicomponent neutral mixtures. At the second stage the linear equation (3.16) with boundary conditions (3.17) and (3.10) is solved, and the electric field distribution is determined. By integrating the latter it is possible to determine the body surface potential that corresponds to a given α. The advantage of such separation over the direct solution of the input problem is evident.

It is important to note that for $\alpha = 0$ the quasi-neutral solution for ionized components becomes exact as $\varepsilon \to 0$. The substitution of the quasi-neutral solution into Eq. (3.16) is thus in essence a linearization with respect to α. Note that in the absence of convection such linearization is valid for any, and not only for small values of parameter ε [13]. The convenience of the introduction of the formal boundary condition (1.6) becomes clear.

As indicated above, the condition of smallness of ε is satisfied in the majority of cases encountered in practice, while the condition of smallness of α means the imposition of a limit on the body surface potential.

4. Example. We shall apply the described method to the one-dimensional problem of electric probe in a three-component gas containing positive singly charged ions of one kind, electrons, and neutral particles of one kind. The degree of gas ionization is assumed weak, so that the effect of collisions between charged particles

is small in comparison with their collisions with neutral particles and can be neglected. The problem of a probe at the stagnation point of a blunt body in a hypersonic stream of weakly ionized gas [14] and, also, the problem of a spherical probe in a quiescent weakly ionized plasma of constant or varying properties belong to this class. We neglect for simplicity the thermal - and baro-diffusions and, also, assume that homogeneous chemical reactions are frozen and that recombination takes place only on body surface. The determining system of equations (1. 1) — (1. 3) is in this case of the form

$$\mathrm{Re}\,\rho v x_j' + r^{-2}(r^2 J_j)' = 0 \quad (j = i, e) \tag{4. 1}$$

$$J_j = \frac{1}{\Delta_{jn}}\left(-x_j' + x_j z_j \frac{E}{T}\right) \quad (j = i, e) \tag{4. 2}$$

$$\varepsilon r^{-2}(r^2 E)' = \rho\,(x_i - x_e) \tag{4. 3}$$

where the subscripts i, e, and n denote ions, electrons, and neutral particles, respectively, and the boundary conditions are (α is a specified quantity)

$$y = 0, \quad x_i = x_e = 0; \quad J_e\Delta_{en} - J_i\Delta_{in} = \alpha \tag{4. 4}$$

$$y \to \infty, \quad x_i \to x_\infty, \quad x_e \to x_\infty$$

Problems of this type were considered in a number of publications [1 — 3].

Let us find the quasi-neutral solution for the concentration and diffusion fluxes of ions and electrons. Assuming that the ratio of resistance coefficients of ion-neutral and electron - neutral particle is constant, for the quasi-neutral concentration of charged particles we obtain the diffusion equation with the obvious boundary conditions

$$\mathrm{Re}\,\rho v x' - r^{-2}(r^2\Delta_a^{-1}x')' = 0, \quad x = x_i = x_e, \quad \Delta_a = (\Delta_{in} + \Delta_{en})/2 \tag{4. 5}$$

$$y = 0, \quad x = 0; \quad y \to \infty, \quad x \to x_\infty$$

Solution of that problem can be expressed in quadratures

$$x = x_\infty \frac{c(y)}{c(\infty)}, \quad x' = x_\infty \frac{\Delta_a}{c(\infty)\,r^2} \exp\left(\int_0^y \mathrm{Re}\,\rho v \Delta_a dt\right) \tag{4. 6}$$

$$c(y) = \int_0^y \frac{\Delta_a}{r^2} \exp\left(\int_0^q \mathrm{Re}\,\rho v \Delta_a dt\right) dq$$

The quasi-neutral diffusion fluxes of ions and electrons are

$$J_i = \frac{1}{2r^2\Delta_{aw}}\left[\left(1 - \frac{\Delta_{en}}{\Delta_{in}}\right)x_w' - \alpha\frac{\Delta_a}{\Delta_{in}}\right] - \frac{x'}{\Delta_a} \tag{4. 7}$$

$$J_e = \frac{1}{2r^2\Delta_{aw}}\left[\left(1 - \frac{\Delta_{in}}{\Delta_{en}}\right)x_w' + \alpha\frac{\Delta_a}{\Delta_{en}}\right] - \frac{x'}{\Delta_a}$$

where the subscript w denotes the respective quantities at the surface of body (the probe).

We pass to the determination of the electric field. In the considered particular case Eq. (3. 16), after the substitution in the right-hand side of formulas (4. 7) for quasi-neutral diffusion fluxes of ions and electrons, assumes the form

$$\frac{\varepsilon}{\rho}E'' - \frac{2x}{T}E = \frac{\Delta_{in} - \Delta_{en}}{\Delta_a}\left(x' - x'_w \frac{\Delta_a}{r^2\Delta_{aw}}\right) + \alpha \frac{\Delta_a}{r^2\Delta_{aw}} \tag{4.8}$$

and the boundary conditions are of the form (3.17) and (3.10). Application of the method of separation to this problem shows that for small ε and α the solution of the nonlinear input problem (4.1)–(4.4) can be obtained in two stages each of which involves the solution of linear equations. At the first stage the linear problem (4.5) is solved and the profile of charged particle concentration is determined. After the determination of concentration by formulas (4.7) it is possible to obtain the profiles of ion and electron fluxes. At the second stage the electric field profile is determined from the solution of the linear boundary value problem (4.8), (3.17), (3.10). By integrating the obtained profile it is possible to determine the potential distribution corresponding to the specified α. The body surface potential relative to infinity is defined by formula

$$\varphi_w = \int_0^\infty E dy \tag{4.9}$$

Let us consider the problem of derivation of the ion and electron characteristics, i.e. the dependence of density of the ion and electron diffusion fluxes to the body surface on its potential.

Exact characteristics are obtained by solving problem (4.1) — (4.4) for various values of α; each solution defines a single point of the ion and electron characteristics. For constructing sections of characteristics corresponding to the region of small α we use the proposed method of separation. For the diffusion fluxes of ions and electrons to the body surface from (4.7) we obtain

$$j_i = 1 + \frac{\alpha}{2x'_w}, \quad j_e = 1 - \frac{\alpha}{2x'_w} \tag{4.10}$$

$$\left(j_i = -\frac{J_{iw}\Delta_{inw}}{x'_w}, \quad j_e = -\frac{J_{ew}\Delta_{enw}}{x'_w}\right)$$

where j_i and j_e are the flux densities to the body surface of ions and electrons, respectively, normalized with respect to their values for $\alpha = 0$.

To obtain the characteristics in explicit form it is necessary to express α in terms of the body surface potential (4.9). For this it is convenient to represent the solution for E in the form of superposition of two functions each of which is independent of α

$$E = E_1 - \frac{\alpha}{2x'_w}E_2 \tag{4.11}$$

where functions E_1 and E_2 are solutions of the following linear equations:

$$\frac{\varepsilon}{\rho}E_1'' - \frac{2x}{T}E_1 = \frac{\Delta_{in} - \Delta_{en}}{\Delta_a}\left(x' - x'_w \frac{\Delta_a}{r^2\Delta_{aw}}\right) \tag{4.12}$$

$$\frac{\varepsilon}{2\rho}E_2'' - \frac{x}{T}E_2 = -\frac{\Delta_a x'_w}{r^2\Delta_{aw}} \tag{4.13}$$

with boundary conditions (3. 17) and (3. 10).

Using formulas (4. 9) and (4. 11) for expressing α in terms of φ_w and substituting the obtained expression into (4. 10) we obtain the sought characteristics in the form

$$j_i = 1 - \frac{\varphi_w - \varphi_1}{\varphi_2}, \quad j_e = 1 + \frac{\varphi_w - \varphi_1}{\varphi_2} \left(\varphi_k = \int_0^\infty E_k dy, \; k = 1, 2\right) \qquad (4.14)$$

where the introduced coefficients φ_1 and φ_2 are independent of α and, consequently, of the body surface potential.

These formulas show in particular that the ion and electron characteristics pass through the point $(\varphi_w = \varphi_1, j_i = j_e = 1)$ to which corresponds α equal zero.

The coefficients φ_1 and φ_2 must be determined by the solution of linear equations (4. 12) and (4. 13). Let us determine these coefficients. The degenerate solution of Eq. (4. 12) is

$$E_1 = \frac{T}{2x} \frac{\Delta_{en} - \Delta_{in}}{\Delta_a} \left(x' - x'_w \frac{\Delta_a}{r^2 \Delta_{aw}}\right)$$

which has no singularities; the indeterminacy at point $y = 0$ can be revealed using the l'Hopital rule. It does not, however, generally satisfy the boundary condition (3. 17). Since the boundary condition is not satisfied by the derivative and not by the function itself and because the discrepancy is of order unity, the boundary layer in the first approximation with respect to ε does not appear, and the degenerate solution is uniformly applicable. Hence

$$\varphi_1 = \frac{\Delta_{en} - \Delta_{in}}{2\Delta_a} \int_0^\infty \frac{T}{x} \left(x' - x'_w \frac{\Delta_a}{r^2 \Delta_{aw}}\right) dy \qquad (4.15)$$

The solution of Eq. (4. 13) may be represented as the sum of solutions of three simpler solutions

$$E_2 = E_3 + E_4 + E_5 \qquad (4.16)$$

$$\frac{\varepsilon}{2\rho} E_3'' - \frac{x}{T} E_3 = -\frac{\Delta_a x'_w}{r^2 \Delta_{aw}} + \frac{T_w x}{Ty}$$

$$\frac{\varepsilon T y}{2\rho x T_w} E_4'' - \frac{y}{T_w} E_4 = \varepsilon E_5'' \left(\frac{1}{2\rho_w x'_w} - \frac{Ty}{2\rho x T_w}\right)$$

$$\frac{\varepsilon}{2\rho_w x'_w} E_5'' - \frac{y}{T_w} E_5 = -1$$

for each of which (3. 17) and (3. 10) represent the boundary conditions. The degenerate solutions of Eqs. (4. 16) for E_3 and E_4 have no singularities, and similarly to the degenerate solution of Eq. (4. 12) are uniformly valid

$$E_3 = \frac{\Delta_a T x'_w}{r^2 \Delta_{aw} x} - \frac{T_w}{y}, \quad E_4 = \left(\frac{y}{T_w} E_5 - 1\right)\left(\frac{T \rho_w x'_w}{\rho x} - \frac{T_w}{y}\right) \qquad (4.17)$$

The last of Eqs. (4. 16) is a nonhomogeneous Airy equation. Its solution which satisfies boundary conditions (3. 17) and (3. 10) is provided by the expression [15]

$$E_5 = \pi T_w \left(\frac{2\rho_w x_w'}{\varepsilon T_w}\right)^{1/3} \left[Gi(z) + \frac{1}{\sqrt{3}} Ai(z)\right], \quad z = y\left(\frac{2\rho_w x_w'}{\varepsilon T_w}\right)^{1/3} \tag{4.18}$$

where $(Gi(z), Ai(z)$ are Airy's functions.

For calculating the potential we shall need the function

$$\theta(z) = \pi \int_0^z \left[Gi(t) + \frac{1}{\sqrt{3}} Ai(t)\right] dt$$

whose asymptotics for large z is obtained by integrating the asymptotics of Airy functions [15]

$$\theta(z) \sim 1.587 + \frac{1}{3}\ln\frac{1}{2} + \ln z - \frac{2}{3}\frac{1}{z^3} + \ldots \tag{4.19}$$

where the constant of integration is represented by the first two terms. It was obtained here by determining function $\theta(z)$ numerically.

Substituting (4.15) into the second of equalities (1.17) and taking into account the asymptotics of Airy functions [15], we find that solution E_4 is uniformly small in comparison with E_5. Substituting the first of equalities (4.17) and (4.18) into the first of formulas (4.16) and integrating the obtained expression, after transformation with allowance for (4.19) we obtain the coefficient φ_2

$$\varphi_2 = \int_0^\infty \left(\frac{\Delta_a T x_w'}{r^2 \Delta_{aw} x} - \frac{T}{ry}\right) dy + T_w \left(1.587 + \frac{1}{3}\ln\frac{\rho_w x_w'}{\varepsilon T_w}\right) \tag{4.20}$$

Note that integrals in (4.15) and (4.20) are independent of ε. Hence for a given distribution of velocity, density, and temperature of gas it is sufficient to calculate these integrals only once.

The applicability of formulas (4.14) is restricted by the assumption of smallness of α. In other words these formulas are applicable when the surface potentials φ_w are fairly close to φ_1, i.e. $|\varphi_w - \varphi_1| \ll \varphi_2$. From the geometrical point of view formulas (4.14) define the linear approximation of ion and electron characteristics at point $(\varphi_1, 1)$.

For obtaining exact ion and electron characteristics in a wide range of body surface potentials problem (4.1) — (4.4) must be solved numerically. But the general form of expected characteristics can be established using a relatively simple asymptotic analysis.

At considerable positive or negative surface potentials and fairly small ε the behavior of characteristics is approximately defined by the following formulas:

$$\varphi_w \to \infty, \quad j_i = 0, \quad j_e = 2; \quad \varphi_w \to -\infty, \quad j_i = 2, \quad j_e = 0$$

In the region of intermediate potentials in the case of homogeneous gas at rest a suitable asymptotic method was developed in [6]. Using an extension of that method to the considered here class of problems it is possible to show that the ion and electron characteristics are odd relative to point $(\varphi_1, 1)$ in the first approximation. In other words, in the first approximation the following identities are valid:

$$j_i(\varphi_1 - \varphi_w) - 1 = -[j_i(\varphi_w - \varphi_1) - 1]$$
$$j_e(\varphi_1 - \varphi_w) - 1 = -[j_e(\varphi_w - \varphi_1) - 1] \qquad (4.21)$$

It follows from this that at point $\varphi_w = \varphi_1$ the second derivatives of functions $j_i(\varphi_w)$ and $j_e(\varphi_w)$ are zero. Hence in the neighborhood of point $(\varphi_1, 1)$ the ion and the electron characteristics are linear within terms of the order of $\varepsilon^{1/3}$. These linear sections are obviously completely defined by formulas (4.14).

The expected ion and electron characteristics are diagrammatically represented in Fig. 1, where the dash lines show the relationships determined by formulas (4.14).

The complete volt-amper characteristic, i.e. the dependence of electric current density at the probe surface on the potential of the latter can be determined form known ion and electron characteristics by the formula

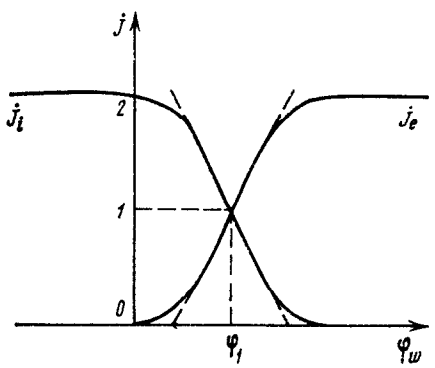

Fig. 1

$$j(\varphi_w) = j_e(\varphi_w) - \frac{\Delta_{enw}}{\Delta_{inw}} j_i(\varphi_w)$$
$$j = J \frac{La_0\Delta_{enw}c(\infty)}{en_0\Delta_{aw}x_\infty}$$

where J is the dimensional density of electric current to the surface. Taking into account formulas (4.21) it is possible to show that the volt-amper characteristic is in the first approximation odd with respect to point $(\varphi_w = \varphi_1,\ j = 1 - \Delta_{enw} / \Delta_{inw})$. In other words the identity

$$j(\varphi_1 - \varphi_w) - \left(1 - \frac{\Delta_{enw}}{\Delta_{inw}}\right) = -\left[j(\varphi_w - \varphi_1) - \left(1 - \frac{\Delta_{enw}}{\Delta_{inw}}\right)\right]$$

is valid within terms of the order of $\varepsilon^{1/3}$

Thus the zero value of parameter α corresponds to the center of symmetry of the volt-amper characteristic.

When interpreting probe measurements the concentration of charged particles in the unperturbed plasma, x_∞ can be determined either by the magnitude of the ion saturation current or by that of the electron saturation current, or by the magnitude of the current which corresponds to the center of symmetry of the volt-amper characteristic. The plasma potential φ_p relative to earth is readily determined by the formula

$$\varphi_p = \varphi_0 - \frac{kT_0}{e}\left(\frac{\Delta_{en} - \Delta_{in}}{\Delta_{en} + \Delta_{in}}\varphi_2 + \varphi_1\right) \qquad (4.22)$$

where φ_0 denotes the potential at the intersection point of the tangent to the volt-amper characteristic drawn through its point of symmetry with the axis of potentials, and φ_p and φ_0 are dimensional quantities.

It is interesting to consider the simplification of formula (4.22) in the case of practical importance of high Reynolds numbers $(\mathrm{Re} \to \infty,\ \mathrm{Re}\,\varepsilon \to 0)$ of the oncoming

stream, when the main input to integrals in formulas (4.15) and (4.20) is provided by the inviscid region. Retaining in these formulas terms of the order of $\mathrm{Re}^{1/2}$ and neglecting those of order unity and $\ln(\mathrm{Re}^{1/2}\varepsilon^{-1})$, we have for this case

$$\varphi_2 = \frac{x_w'}{\Delta_{aw}} \int_0^\infty \frac{T\Delta_a}{r^2}\, dy, \quad \varphi_1 = \frac{\Delta_{in} - \Delta_{en}}{\Delta_{in} + \Delta_{en}}\, \varphi_2$$

Formula (4.22) now assumes the simple form: $\varphi_p = \varphi_0$. Thus in the case of high Reynolds numbers the potential of plasma is equal to that determined by the intersection point of the tangent to the volt-amper characteristic drawn through the latter center of symmetry with the axis of potentials.

All of the above reasoning is valid also in the presence of homogeneous chemical reactions, except that in this case the right-hand side of the diffusion equation (4.5) contains an inhomogeneous source term, and formula (4.6) becomes inapplicable for quasi-neutral concentration of charged particles. There are no fundamental difficulties in taking into account thermal-and baro-diffusions.

The expounded here method was used in [13] for the determination of electrical effects in the neighborhood of a conducting sphere in a quiescent two-temperature weakly ionized gas. The formulas derived there can be used for determining the temperature of electrons by the measured volt-amper characteristic.

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